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TRANSIENT FILM CONDENSATION ON A VERTICAL SURFACE IN A POROUS MEDIUM

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NOMENCLATURE

A	constant defined in equation (11a)
B	constant defined in equation (11b)
C	constant defined in equation (16)
C_p	specific heat at constant pressure
g	gravitational acceleration
h_{fg}	heat of vaporization
Ja	Jakob number
K	permeability of the porous medium
k_c	thermal conductivity of the fluid-filled porous medium
L	characteristic length
$\dot{m}$	mass flux of condensate
Nu_x	local Nusselt number
Pc	condensation parameter
Pc_x	local condensation parameter
T	temperature
t	time
u	Darcy's velocity in the x-direction
X	dimensionless coordinate in the x-direction defined in equation (13c)
x	coordinate along the vertical plate
y	coordinate perpendicular to the plate.

Greek symbols

α	equivalent thermal diffusivity
Δ	dimensionless film thickness defined in equation (13a)
δ	condensate film thickness
θ	dimensionless temperature defined in equation (12b)
μ	viscosity of the condensate
ξ	dimensionless distance defined in equation (12a)
ρ	density of fluid
σ	heat capacity ratio defined in equation (3)
τ	dimensionless time defined in equation (13b)
ϕ	porosity.

Subscripts

c	composite quantities
s	saturation condition
v	vapor phase
L	liquid phase
w	condition at the wall
∞	condition at infinity.

INTRODUCTION

IN A PREVIOUS paper, Cheng [1] has considered the problem of steady condensation of a liquid film flowing downward along a cooled inclined surface in a porous medium that is filled with a dry saturated vapor. In this note, consideration will be given to the transient problem of the growth of the condensate and the associated heat transfer characteristics in a porous medium adjacent to a vertical plate when its temperature is suddenly cooled below the saturation temperature corresponding to its pressure. On the basis of the Karman–Pohlhausen integral method, it will be shown that the governing equation for the growth of the condensate is a first-order hyperbolic equation. An analytical solution will be obtained based on the method of characteristics [2, 3]. It will be shown that at small times, the problem becomes that of the growth of a liquid film with uniform thickness. After the thickness of the film reaches that of steady state, the heat transfer mode changes abruptly from transient one-dimensional (1-D) film condensation to steady two-dimensional (2-D) film condensation. It will be shown that the approximate solution for the temperature distribution, surface heat flux and film thickness obtained from the present note for steady state film condensation agree well with the similarity solution obtained in a previous paper [1].

ANALYSIS

As in the previous paper [1], it is assumed that (a) the condensate and the dry saturated vapor are separated by a distinct interface, (b) the properties of the porous medium as well as those of the condensate and the vapor are constant, and (c) boundary layer approximations are applicable. Under these assumptions, the energy equation with the aid of the continuity equation, can be integrated across the liquid film (with thickness δ) to give

$$(\rho C_p)_c \frac{\partial}{\partial t}\int_0^\delta (T-T_s)\,dy + (\rho C_p)_L \frac{\partial}{\partial x}\int_0^\delta u(T-T_s)\,dy = k_c\left[\frac{\partial T}{\partial y}(x,\delta,t) - \frac{\partial T}{\partial y}(x,0,t)\right], \quad (1)$$

where x and y are the coordinates along and perpendicular to the vertical plate, T and T_s are the temperature and the saturation temperature of the condensate, $(\rho C_p)_c$ and k_c are the heat capacity and the thermal conductivity of the fluid-filled porous medium defined as $(\rho C_p)_c = (1-\phi)(\rho C_p)_m + \phi(\rho C_p)_L$ and $k_c = (1-\phi)k_m + \phi k_L$ where ϕ is the porosity and the subscripts 'm' and 'L' denote the quantities associated with the porous medium and the saturated liquid, respectively. Equation (1) is coupled with the Darcy law through the Darcy velocity in the x-direction, which is given by [1]

$$u = \frac{K}{\mu}(\rho_L - \rho_v)g, \quad (2)$$

where K is the permeability of the porous medium, g is the gravitational acceleration, and ρ_v and ρ_L are the densities of the saturated vapor and liquid. Substituting equation (2) into equation (1) yields

$$\sigma\frac{\partial}{\partial t}\int_0^\delta (T-T_s)\,dy + \frac{K}{\mu}(\rho_L-\rho_v)g\frac{\partial}{\partial x}\int_0^\delta (T-T_s)\,dy = \alpha_c\left[\frac{\partial T}{\partial y}(x,\delta,t) - \frac{\partial T}{\partial y}(x,0,T)\right], \quad (3)$$

where $\sigma \equiv (\rho C_p)_c/(\rho C_p)_L$ and $\alpha_c = k_c/(\rho C_p)_L$. Equation (3) is to be solved subject to the initial condition

$$T(x, y, 0) = T_s, \tag{4}$$

with the boundary condition at the wall given by

$$T(x, 0, T) = T_w, \quad t > 0, \tag{5}$$

and the boundary conditions at the vapor–liquid interface $y = \delta(x, t)$ given by

$$T(x, \delta, t) = T_s, \tag{6}$$

and

$$\dot{m}h_{fg} = k_c\left(\frac{\partial T}{\partial y}\right)_{y=\delta}, \tag{7}$$

where h_{fg} is the latent heat of phase change and $\dot{m}$ is the mass flux of the condensate which is related to the film thickness by

$$\dot{m} = \frac{\partial}{\partial x}\int_0^\delta \rho_L u \, dy + \rho_L \frac{\partial \delta}{\partial t}. \tag{8}$$

Equation (8) can be obtained by considering the conservation of mass in an elementary control volume across the film thickness. Substituting equations (2) and (8) into equation (7) yields

$$k_c\left(\frac{\partial T}{\partial y}\right)_{y=\delta} = \rho_L h_{fg}\left[\frac{K(\rho_L - \rho_v)g}{\mu}\frac{\partial \delta}{\partial x} + \frac{\partial \delta}{\partial t}\right]. \tag{9}$$

With the aid of equation (9), equation (3) can be expressed in the following dimensionless form

$$\left[A + \frac{1}{\sigma Ja}\right]\frac{\partial \Delta}{\partial \tau} + \left[A + \frac{1}{Ja}\right]\frac{\partial \Delta}{\partial X} = -\frac{B}{\Delta}, \tag{10}$$

where

$$A = \int_0^1 \theta(\xi)\, d\xi \quad \text{and} \quad B = \theta'(0), \tag{11a,b}$$

$$\xi = y/\delta, \quad \theta(\xi) = (T - T_s)/(T_w - T_s), \tag{12a,b}$$

and

$$\Delta = \delta L, \quad \tau = \alpha_c t/\sigma L^2, \quad X = x/LPc. \tag{13a,b,c}$$

The dimensionless parameter Ja in equation (10) is the Jakob number defined by $Ja = C_{pL}(T_s - T_w)/h_{fg}$ (with C_{pL} denoting the specific heat of the condensate at constant pressure) which is a measure of the degree of subcooling of the cold plate. In equation (13c) Pc is a condensation parameter in a porous medium defined as $Pc = K(\rho_L - \rho_v)gL/\mu_L\alpha_c$ (with L denoting the length of the plate) which bears some similarity to the Rayleigh number in free convection in a porous medium. Equation (10) will now be solved subject to the initial condition

$$\tau = 0, \quad \Delta = 0, \tag{14}$$

and the boundary condition

$$X = 0, \quad \Delta = 0. \tag{15}$$

We now assume a temperature profile of the form

$$\theta(\xi) = 1 - C\xi + (C-1)\xi^2, \tag{16}$$

which satisfies the boundary conditions (4) and (5), i.e.

$$\theta(0) = 1 \quad \text{and} \quad \theta(1) = 0. \tag{17a,b}$$

The constant C in equation (16) is the negative dimensionless temperature gradient at the wall which remains to be determined from equation (9) after equation (10) has been solved. Substituting equation (16) into equations (11a) and (11b) gives

$$A = (4 - C)/6 \quad \text{and} \quad B = -C. \tag{18a,b}$$

From the method of characteristics, equation (10) with equations (18a) and (18b) is equivalent to

$$\frac{dX}{\left[\frac{4-C}{6} + \frac{1}{Ja}\right]} = \frac{d\tau}{\left[\frac{4-C}{6} + \frac{1}{\sigma Ja}\right]} = \frac{\Delta\, d\Delta}{C}, \tag{19}$$

which has the characteristics

$$dX = \frac{\left[\frac{4-C}{6} + \frac{1}{Ja}\right]d\tau}{\left[\frac{4-C}{6} + \frac{1}{\sigma Ja}\right]}. \tag{20}$$

On each characteristic, Δ is related by

$$\Delta\, d\Delta = \frac{C\, d\tau}{\left[\frac{4-C}{6} + \frac{1}{\sigma Ja}\right]}, \tag{21}$$

or

$$\Delta\, d\Delta = \frac{C\, dX}{\left[\frac{4-C}{6} + \frac{1}{Ja}\right]}, \tag{22}$$

depending on whether the characteristics intersect with the τ or the X-axis. It follows from equation (20) that the limiting characteristic is

$$\tau = \left[\frac{\frac{4-C}{6} + \frac{1}{\sigma Ja}}{\frac{4-C}{6} + \frac{1}{Ja}}\right]X. \tag{23}$$

As we shall see, the value of C is positive with a magnitude less than 2. Since the values of σ and Ja are all positive, the value of the square bracket in equation (23) is also positive. Thus, if X is the abscissa and τ is the ordinate, equation (23) is a straight line passing through the origin of the X–τ plane, and divides the first quadrant of the plane into two regions: the lower region with Δ given by equation (21) and the upper region with Δ given by equation (22). It will be shown that the lower region corresponds to transient film condensation while the upper region corresponds to steady film condensation.

Solution for transient film condensation

Integrating equation (21) with the initial condition equation (14) gives

$$\Delta = \sqrt{\left[\frac{2C\tau}{\frac{4-C}{6} + \frac{1}{\sigma Ja}}\right]}, \tag{24}$$

or

$$\delta = \sqrt{\left[\frac{2C}{\frac{4-C}{6} + \frac{1}{\sigma Ja}}\right]}\left(\frac{\alpha_c t}{\sigma}\right)^{1/2}, \tag{25}$$

which shows that the boundary layer thickness grows as the square root of time and independent of x. To determine the value of C in equations (24) and (25), we substitute equation (25) into equation (9) and use equation (16) to obtain

$$C_{1,2} = [3(\sigma Ja + 2) \pm \sqrt{(9(\sigma Ja + 2)^2 - 4\sigma Ja(2\sigma Ja + 3))}]/\sigma Ja, \tag{26}$$

where C_1 and C_2 correspond to the positive and negative signs in front of the square root term in equation (26). Equation (26) shows that C depends on σJa. A simple calculation reveals that C_1 would give physically impossible results for temperature profiles and thus is inadmissible. The value of C_2 as

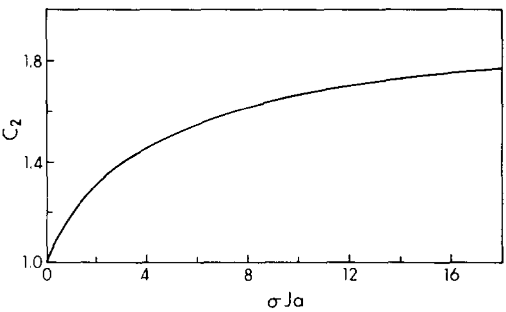

FIG. 1. C_2 vs σJa.

a function of σJa is plotted in Fig. 1 where the value of C_2 varies from 1 to 2 as σJa varies from 0 to ∞. The surface heat flux is given by

$$q_w = -k_c\left(\frac{\partial\phi}{\partial y}\right)_{y=0} = \frac{k_c(T_w - T_s)C}{\delta}. \quad (27)$$

Substituting equation (25) into equation (27) gives

$$q_w = k_c(T_w - T_s)\Big/\left(C\left(\frac{4-C}{6} + \frac{1}{\sigma Ja}\right)\right)\left[\frac{\sigma}{2\alpha_c t}\right]^{1/2}, \quad (28)$$

where $C = C_2$ in equations (27) and (28). Equations (25) and (28) are similar in form to the approximate solution given by Goodman [4] for transient 1-D heat conduction in a semi-infinite solid with phase change when the surface temperature is suddenly changed from T_s to T_w. Goodman [4] has shown that his approximate solution agrees well with the exact solution obtained by Carslaw and Jaeger [5]. Equations (24)–(26) and (28) are valid for the region where

$$\tau < \left[\frac{\dfrac{4-C_2}{6} + \dfrac{1}{\sigma Ja}}{\dfrac{4-C_2}{6} + \dfrac{1}{Ja}}\right] X, \quad (29a)$$

or

$$t < \left[\frac{\dfrac{4-C_2}{6} + \dfrac{1}{\sigma Ja}}{\dfrac{4-C_2}{6} + \dfrac{1}{Ja}}\right] \frac{\sigma\mu_L x}{K(\rho_L - \rho_v)g}, \quad (29b)$$

Solution for steady film condensation

Integrating equation (22) with the boundary condition equation (15) yields

$$\Delta = \sqrt{\left[\frac{2CX}{\dfrac{4-C}{6} + \dfrac{1}{Ja}}\right]}, \quad (30a)$$

or

$$\frac{\delta\sqrt{(Pc_x)}}{x} = \sqrt{\left[\frac{2C}{\dfrac{4-C}{6} + \dfrac{1}{Ja}}\right]}, \quad (30b)$$

where $Pc_x = K(\rho_L - \rho_v)gx/\mu_L\alpha_c$ is the local film condensation parameter, and $\delta\sqrt{(Pc_x)}/x$ is the dimensionless film thickness parameter defined in ref. [1]. To determine the value of C in equations (30a) and (30b), we substitute equation (30b) into equation (9) to give

$$C_{3,4} = [3(Ja+2) \pm \sqrt{(9(Ja+2)^2 - 4Ja(2Ja+3))}]/Ja. \quad (31)$$

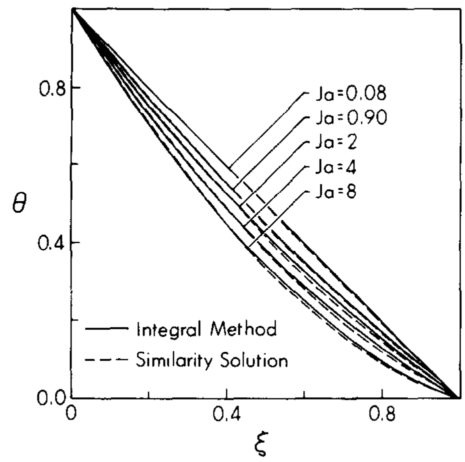

FIG. 2. Steady state dimensionless temperature profiles in the liquid film.

Note that equation (31) is similar to equation (26) except that σJa is now replaced by Ja. Since C_3 would give physically inadmissible results, only C_4 will be used.

With C_4 determined from equation (31), the dimensionless temperature profile given by equation (16) and the dimensionless film thickness parameter given by equation (30b) can now be plotted as a function of Ja. These results are presented as solid lines in Figs. 2 and 3, respectively. Since they are independent of time, they are the steady state solutions. The results from the similarity solutions [1] are also plotted as dashed lines on the same figures for comparison. It is shown that for steady film condensation, the results from the integral method obtained in the present paper agree well with the exact similarity solution obtained in a previous paper.

Substituting δ from equation (30b) in equation (27) gives the heat flux for steady film condensation as

$$q_w = k_c(T_w - T_s)\left\{\sqrt{\left(\frac{C}{2}\left(\frac{4-C}{6} + \frac{1}{Ja}\right)Pc_x\right)}\right\}\Big/x, \quad (32a)$$

or

$$\frac{Nu_x}{\sqrt{Pc_x}} = \left[\frac{C}{2}\left(\frac{4-C}{6} + \frac{1}{Ja}\right)\right]^{1/2}, \quad (32b)$$

where $Nu_x = q_w x/k_c(T_w - T_s)$ and C is given by C_4 in equation (31). Note that equations (32a) and (32b) depend only on Ja

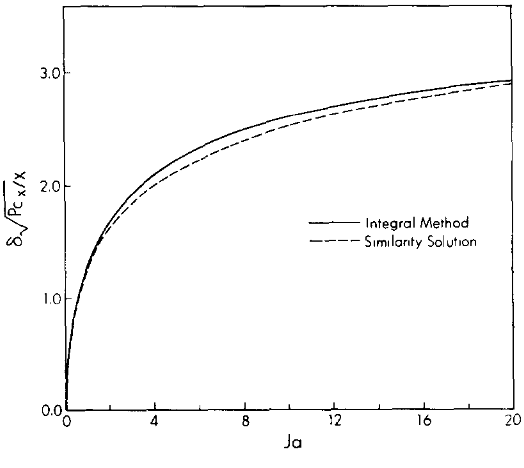

FIG. 3. Steady state film thickness.

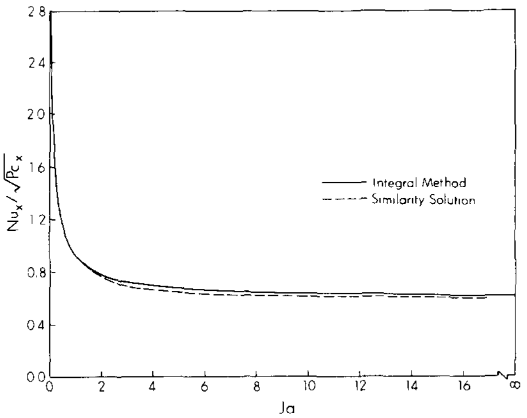

FIG. 4. Steady state heat transfer results.

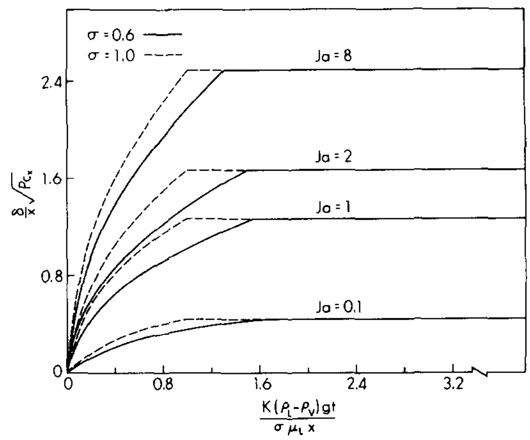

FIG. 5. The growth of the liquid film thickness with time.

and are independent of σ, since σ is associated with the transient term in equation (3). Equation (32b) as a function of Ja is plotted as a solid line in Fig. 4, where it is shown that the results agree well with the similarity solution. Equations (30)–(32) are valid for steady film condensation where

$$t > \left[\frac{\dfrac{4-C_2}{6}+\dfrac{1}{\sigma Ja}}{\dfrac{4-C_2}{6}+\dfrac{1}{Ja}}\right]\frac{\sigma\mu_L x}{K(\rho_L-\rho_v)g}. \tag{33}$$

The growth of the film thickness as given by equation (25) for the transient period and equation (30b) for the steady state is plotted in Fig. 5 for selected values of Ja and σ. It is shown that the film thickness continues to grow with time until steady state where the film thickness remains constant thereafter.

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